Satellites - Trilateration

Objectives

Students Will:

- Determine an unknown location given the signal readings of distance from three satellites
- Use formal geometric constructions to determine an unknown location
- Derive the equation of a circle given the center and radius

Suggested Grade Level

10th

Subject Areas

Mathematics

Timeline

50-60 minutes

Standards

- **CCSS.MATH.CONTENT.HSG.CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
- **CCSS.MATH.CONTENT.HSG.GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

21st Century Essential Skills

- critical thinking/problem solving, analysis, interpretation, collaboration and teamwork, communication, information literacy, organizing concepts, carrying out investigations, constructing explanations, designing solutions, obtaining/evaluating/communicating ideas

Background

Do you ever wonder how your phone can know your exact location? How it can tell you where the closest fast food restaurant is? Or track how many steps you take in a day?

Your phone has a built in GPS system. The Global Positioning System (GPS) consists of a network of 24 active satellites and 8 backup satellites orbiting in medium Earth orbit (MEO), at an altitude of approximately 12,550 miles. GPS receivers use trilateration to determine locations around the globe. GPS receivers determine distance by analyzing radio signals transmitted from satellites and determining the time it takes for the signal to travel from the satellite to the receiver.

For our activity, we will demonstrate finding an unknown location in two dimensions. Students will be given three GPS satellites with a known position. The distance from one GPS to the unknown location forms a circle (with the satellite as the center and the distance as the radius). The unknown location must fall somewhere along that circle. If we then have two more satellites with known distances, we can create circles from those measurements as well. Where the three circles intersect is the unknown location.

In reality, trilateration works in three dimensions, involves using spheres, and requires four satellite signals to determine a unique location. A visual representation of the way GPS works in three dimensions is attached to this lesson.

Vocabulary

Geometric constructions, trilateration, satellite

Materials

- Pixelated map of the world (attached)
- Information Sheet (attached)
- Satellite Coordinates Sheet (attached)
- Pencil or fine-tipped marker/pen
- String
- Ruler
- Compass

Lesson

1. Distribute copies of map and Information Sheet to groups of students (one per group).
2. Each group chooses a location on the map (this needs to be kept secret from other groups).

3. Students measure the distance from their chosen location to three satellites (measure in centimeters). For consistency, have students measure from the illustrated point on each satellite.

4. Complete Part A of the Information Sheet for their distances. Students should wait for teacher instructions before moving on to Part B of the Information Sheet.
5. Have groups trade Information Sheets.
6. With the information provided on the Information Sheets, students use the measurements to cut the string into each of the lengths provided.
7. Measure from the denoted satellites with the string to find the location chosen. Students can construct circles or arcs with the string held at the determined length to see where the three intersect.
8. Groups should verify (silently) their finding with the other group to determine if they were correct.
9. Repeat steps 5-8 so that groups have multiple opportunities to practice (approximately 20 total minutes).
10. The next portion of the lesson will have students create equations of circles given the center and radius of the circle. Tell students that they are going to imagine their map is on a coordinate plane. Each satellite will be assigned a pair of coordinates (x, y), according to the list attached (Satellite Coordinates). These coordinates represent the centers of the circles.
11. Students are to select the coordinates for the three satellites that they originally used when determining their secret location.
12. Students will derive the equations of the circles using their satellite coordinates and its corresponding radius. They should end up with three equations for their three circles.
    a. Teacher option: you can have students graph each of the satellite coordinates on a centimeter coordinate plane if they need a visual option. Students can plot their coordinates on a coordinate plane and then use the corresponding radius (length they measured from the satellite to the secret location) to create a circle on the coordinate plane.

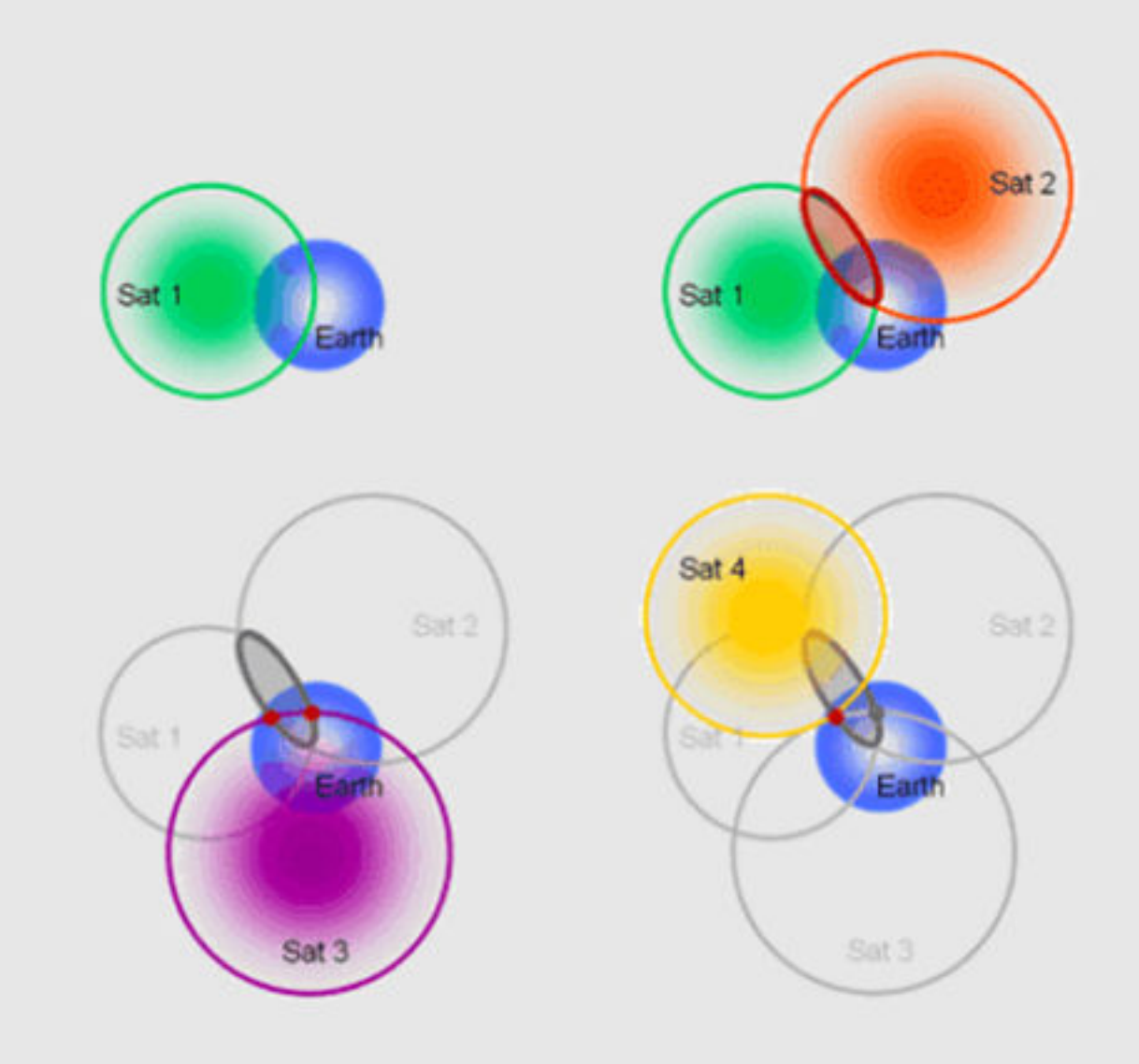
Extensions

- Have students solve the system of equations to find the exact coordinates of their secret location.

Resources

- Gps satellite png 3 " PNG Image png for Free Download. (n.d.). Retrieved April 15, 2020, from https://dlpng.com/png/4089293
- How GPS Works. (2014). Retrieved April 14, 2020, from https://www.gtav.asn.au/documents/item/422
- Raybek. (2018, April 12). Dots world map stock illustration. Illustration of abstract - 114288248. Retrieved April 15, 2020, from https://www.dreamstime.com/colored-world-map-dots-white-background-blue-north-america-red-africa-yellow-asia-green-south-america-orange-australia-image114288248
- Schmandt, M. (n.d.). Ch. 2: Input. Retrieved April 16, 2020, from https://giscommons.org/chapter-2-input/

Visual for How Satellites Determine Location in Three Dimensions



Information Sheet

Part A

1. **Choose a "secret" location (signified by a dot) on the map. Circle this location on your own map, but do not show your location (or map) to other groups.**

2. **Choose three satellites from which other groups will determine your secret location.** Our three satellites are: ____, ____, & ____.

3. **Using a ruler, measure the distance from each of these satellites to your secret location (measure in centimeters) and complete the information below:**
   - Satellite ____; Distance from secret location: _____
   - Satellite ____; Distance from secret location: _____
   - Satellite ____; Distance from secret location: _____

Part B – STOP (wait for teacher instructions)

4. **Trade Information Sheets with another group (do NOT trade or show your map!).**
5. **Use the other groups' satellite and distance information to determine their secret location.**
   a) Cut your string to one of the distance measurements the other group provided above.
   b) On your map, place one end of the string on the satellite and pull the string until taut.
   c) Use a pencil to draw a circle (or arc) to show the possible locations where the location could be along the radius. (The satellite should be the center of that circle).
   d) Repeat steps a - c for the other two given distance measurements.
   e) Once you've completed circles (or arcs) for the three measurements, can you find where the secret location is?

Satellite Coordinates

A (-5, 4)	**M (4, -5)**
B (-4, 5)	**N (3, -5)**
C (-2, 5)	**O (1, -5)**
D (-1, 5)	**P (0, -5)**
E (0, 5)	**Q (-1, -5)**
F (2, 5)	**R (-3, -5)**
G (4, 5)	**S (-4, -5)**
H (5, 4)	**T (-5, -4)**
I (5, 3)	**U (-5, -2)**
J (5, 0)	**V (-5, 0)**
K (5, -2)	**W (-5, 2)**
L (5, -4)	**X (-5, 3)**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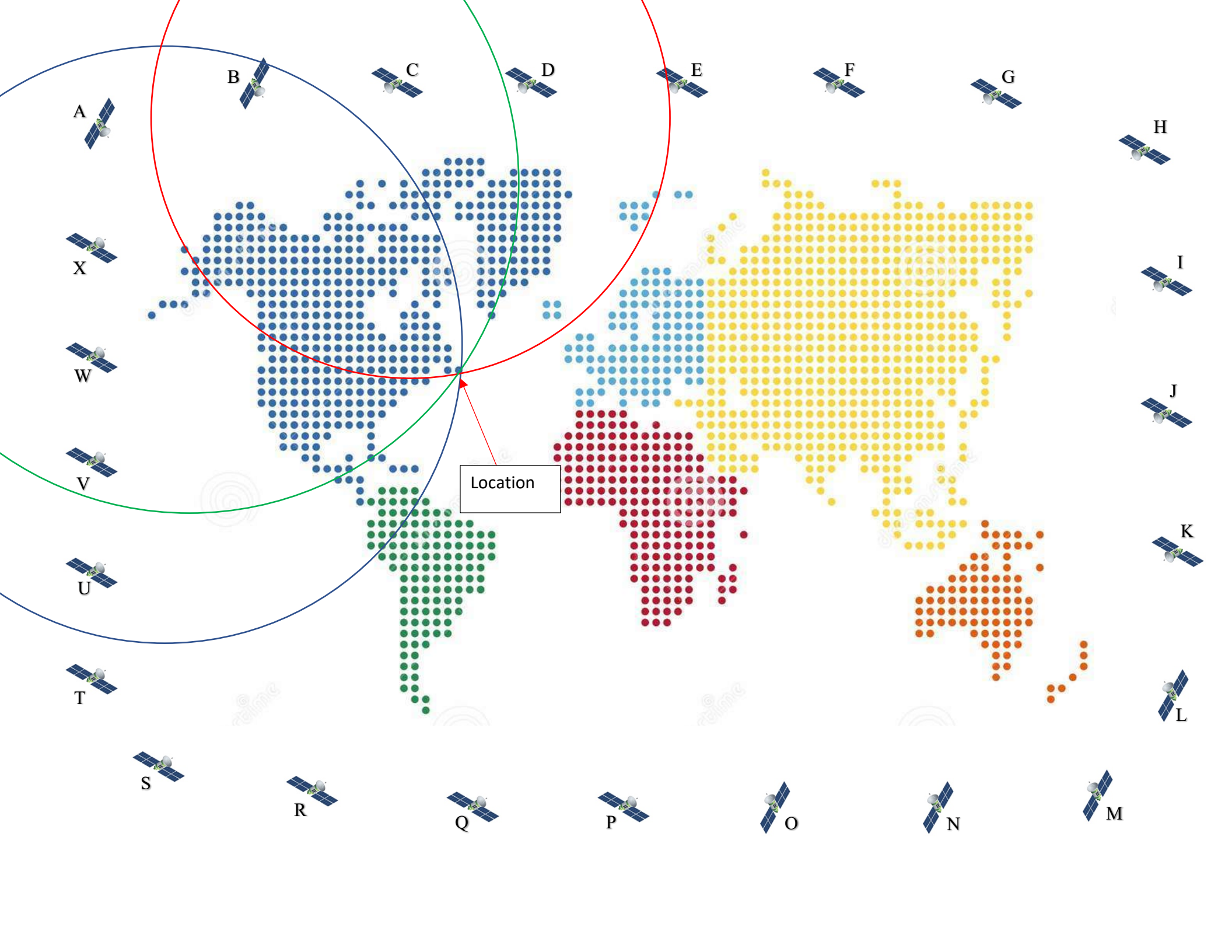
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